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Third Semester B.E. Degree Examination, December 2011
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Define set, power set, complement of a set. Give one example for each. **(06 Marks)**

b. One hundred students were asked whether they had taken courses in any of the three areas, Kannada, English and Hindi. The results were: 45 had taken Kannada, 38 had taken English, 21 had taken Hindi, 18 had taken Kannada and English, 9 had taken Kannada and Hindi, 4 had taken Hindi and English and 23 had taken no course in any of the areas. Draw a Venn diagram that shows the result of the survey and determine the number of students, who had taken course in exactly, i) One of the areas and ii) Two of the areas. **(08 Marks)**

c. If A and B are events in a finite sample space E and $A \subset B$, then show that, $P(A) \leq P(B)$. **(06 Marks)**

2 a. Define logical connectives conjuction and disjunction, with the corresponding truth table. **(06 Marks)**

b. Construct the truth table for $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$. **(06 Marks)**

c. Test the validity of the following argument.
If I like mathematics, than I will study
Either I study or I fail
Therefore if I fail, then I do not like mathematics. **(08 Marks)**

3 a. Let p(x) denotes the sentence "x + 2 > 5". State whether or not p(x) is a proportional function on each of the following sets:
i) N, the set of +ve integers ii) C, the set of complex numbers. **(04 Marks)**

b. Negate the following statements:
i) $\forall x\, p(x) \wedge \exists y\, q(y)$ ii) $\exists x\, p(x) \vee \forall y\, q(y)$. **(04 Marks)**

c. Define rule of universal specification and rule of universal generalization. Also write their symbolical notation forms. **(06 Marks)**

d. Prove by Mathematical Induction that,
$$1 + 2 + 3 + \ldots\ldots\ldots\ldots + n = \frac{n(n+1)}{2}.$$ **(06 Marks)**

4 a. Prove that $2^n \geq n^2$ for $n \geq 4$. **(06 Marks)**

b. What is meant by a recursively defined function? Calculate 4! Using the recursive function. **(06 Marks)**

c. Define the Cartesian product of two sets. Let A = {2, 3, 4} and B {4, 5}. Then find
i) A x B ii) B x V iii) B^2 iv) A^2 **(08 Marks)**

PART – B

5 a. Let R be the relation from A = {1, 2, 3,4} to B {x, y, z} defined by R = {(1, y), (1, z), (3, y), (4, x), (4, z)}. Determine the domain, range and inverse relation of R. **(05 Marks)**

b. Define the one – one and onto function. Give one example for each. **(04 Marks)**

c. If $f : A \rightarrow B$ with $A_1, A_2 \subseteq A$. Then prove that i) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$
ii) $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$. **(06 Marks)**

d. State the Pigeonhole principle. Give one suitable example which satisfies the principle. **(05 Marks)**

6 a. Define composition function, with an example. (04 Marks)

b. Explain a subgraph of a directed graph G (V, E). Give one example. Draw the directed graph of G (V, E), where V (G) = {A, B, C, D} and E (G) = {(A, B), (A, C), (B, C), (B, D), (C, C), (D, B)} (06 Marks)

c. Let A = {1, 2, 3, 4, 12}. Consider the partial order of divisibility on A. That is, if a, b $\in$ A, a $\leq$ b iff a/b. Draw the Hasse diagram of the poset (A, $\leq$). (04 Marks)

d. Define the equivalence relation. Prove an equivalence relation by considering one example. (06 Marks)

7 a. Let G be the group of real numbers under addition and G^1 be the group of +ve numbers under multiplication. Show that the mapping f: G $\rightarrow$ G^1, defined by f(a) = 2^a is homomorphism and isomorphism. (06 Marks)

b. State and prove Lagrange's theorem. (08 Marks)

c. A (3, 8) encoding function e: $B^3 \rightarrow B^8$ defined by
e (000) = 0000 0000
e (001) = 1011 1000
e (010) = 0010 1101
e (011) = 1001 0101
e (100) = 1010 0100
e (101) = 1000 1001
e (110) = 0001 1100
e (111) = 0011 0001

Find how many errors will e detect. (06 Marks)

8 a. If α = 001110 and β = 011011 find:
   i) Weight of α and β
   ii) Distance between α and β. (10 Marks)

b. Given a ring (R, +, •), for all a, b $\in$ R. Prove the following:
   i) $-(-a) = a$
   ii) $a(-b) = (-a) = -(ab)$
   iii) $(-a)(-b) = ab$. (10 Marks)

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